ALGEBRA (POLINOMI, RELAZIONI RADICI-COEFFICIENTI)

POLINOMIO: $P(x) = a_m \cdot x^m + a_{m-1} \cdot x^{m-1} + \dots + a_1 \cdot x + a_0$

$a_m \neq 0$ $\quad \deg(P) = m$

$$\boxed{\begin{array}{l} x^6 + 2x^5 - 3x^3 + x^2 - 9 \\ p(-1) = 1 - 2 + 3 + 1 - 9 \end{array}}$$

$P(0) = a_0$

$P(1) = a_m + \dots + a_0$

$P(x)$ e $q(x)$

$\deg = 3$ $\quad \deg = 3$ $\quad \deg(p+q) \leq 3$

$\deg(p \cdot q) = 6$

$\deg(p(p(x)))$ $\quad p^2(x)$

$(x^3)^3 = x^9$

ex

$$P(x) = (x+1)^5 + (x-1)^{11} - 30\cdot(x-2)^4\cdot x^{29} \qquad \deg = 33$$

$$P(0) = 1^5 + (-1)^{11} = 1 - 1 = 0$$

$$P(1) = (2)^5 + 0^{11} - 30(-1)^4\cdot 1 = 32 - 30 = 2$$

$$\frac{P(1) + P(-1)}{2} = \left(\text{PARI} + \cancel{\text{DISP}}\right) + \left(\text{PARI} - \cancel{\text{DISP}}\right) = \frac{2\cdot \sum \text{PARI}}{2}$$

$$\frac{P(1) - P(-1)}{2} = \sum \text{DISPARI}$$

DIVISIONE TRA POLINOMI

$f(x)$ DIVIDENDO

$g(x)$ DIVISORE $\neq 0$

$q(x)$ QUOZIENTE

$R(x)$ RESTO

$\forall x \in \mathbb{R}$

$$f(x) = g(x) \cdot q(x) + R(x)$$

$$\deg(R) < \deg(g)$$

SE $g(x) = x - \alpha$

ALLORA $R(x) = R \in \mathbb{R}$

$$f(x) = (x-\alpha) \cdot q(x) + R$$

TEO DEL RESTO

$x=\alpha \to f(\alpha) = \cancel{(\alpha-\alpha) \cdot q(\alpha)} + R$

$7 \cdot (x^2)^8 \cdot x - 4 \cdot (x^2)^7 \cdot x = 7x - 4x = 3x$

ex: $f(x) = 7 \cdot x^{17} - 4 \cdot x^{15}$

TROVA IL RESTO DELLA DIVISIONE

$x^2 \equiv 1$ $\quad g(x) = x+1 = x-(-1)$

$\alpha = -1$ $\quad R = f(-1) = 7 \cdot (-1)^{17} - 4 \cdot (-1)^{15} =$

$= -7 + 4 = -3 = f(-1)$

e SE... $g(x) = x^2 - 1$?

$= (x+1)(x-1)$

$R(x) = 3x$ $\quad b = 0$, $a = 3$

$f(x) = (x+1)(x-1) \cdot q(x) + (ax+b)$

$x = -1 \rightarrow f(-1) = 0 \cdot (-2) \cdot q(-1) + (-a+b) \longrightarrow -a+b = -3$

$x = 1 \rightarrow f(1) = 0 + (a+b) \longrightarrow a+b = 7-4 = 3$

$$x^6 + \frac{1}{x^6} = 23 \qquad x^9 + \frac{1}{x^9} = ? \qquad x \in \mathbb{R}^+$$

$$x^3 = t \in \mathbb{R}^+ \qquad t^2 + \frac{1}{t^2} = 23 \qquad t^3 + \frac{1}{t^3} = ?$$

$$\left(t + \frac{1}{t}\right)^2 = t^2 + \frac{1}{t^2} + 2 = 25 \rightsquigarrow t + \frac{1}{t} = 5$$

$$\boxed{(A+B)^3 = A^3 + B^3 + 3AB(A+B)}$$

$$\left(t + \frac{1}{t}\right)^3 = \boxed{t^3 + \frac{1}{t^3}} + 3 \cdot 1 \left(t + \frac{1}{t}\right)$$

$$t^3 + \frac{1}{t^3} = 5^3 - 3 \cdot 5 = 110$$

$$\begin{cases} a+b=20 \\ a\cdot b=91 \end{cases}$$

NON È PRIMO

$$a^2+b^2=(a+b)^2-2(ab)=20^2-2\cdot 91=218$$

$a=13 \quad b=7 \qquad 169+49=218$

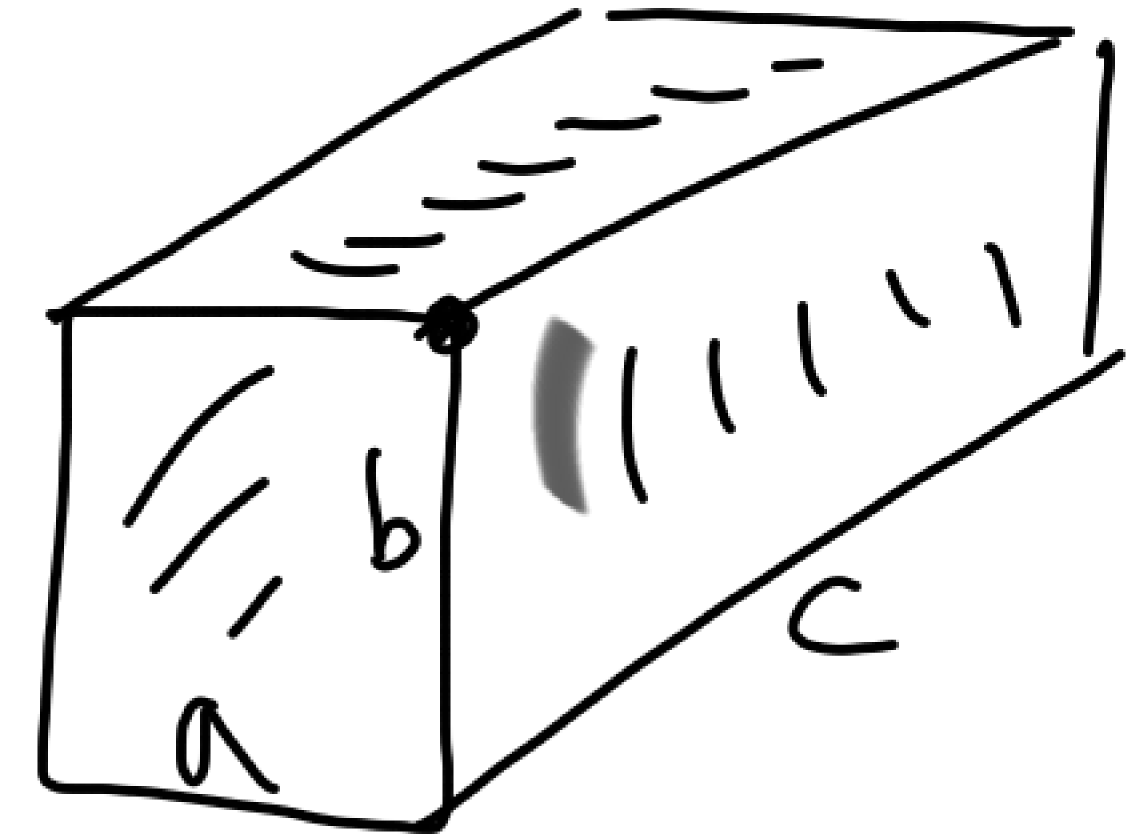
$$\begin{cases} a+b=20 \\ a\cdot b=144 \end{cases}$$

$\Delta<0 \rightarrow$

$$a^2+b^2=(a+b)^2-2\cdot(ab)=400-288=112$$

ANCHE SE NON TROVO a e b

PARALLELEPIPEDO RETTANGOLO



$ab = 429 = 3 \cdot 11 \cdot 13$

$bc = 364 = 2^2 \cdot 7 \cdot 13$

$ac = 231 = 3 \cdot 11 \cdot 7$

$$V = abc = \sqrt{(abc)^2} = \sqrt{ab \cdot bc \cdot ac}$$

$$= \sqrt{(2 \cdot 3 \cdot 7 \cdot 11 \cdot 13)^2}$$

$$= 2 \cdot 3 \cdot \boxed{7 \cdot 11 \cdot 13} = 1001$$

$$= 6006$$

MONICO deg = 2

α, β RADICI

$$P(x) = X^2 + \underbrace{a_1} \cdot X + \underline{\underline{a_0}} = (X-\alpha)(X-\beta)$$

$$= X^2 - \beta X - \alpha X + \alpha\beta$$

$$= X^2 \underbrace{- (\alpha+\beta)} X + \underline{\underline{\alpha \cdot \beta}}$$

$$a_1 = -(\alpha+\beta)$$

$$a_0 = \alpha \cdot \beta \longrightarrow \begin{cases} \alpha+\beta = -a_1 \\ \alpha \cdot \beta = a_0 \end{cases}$$

ex: $P(x) = x^3 - 5x^2 + 3x + 8$

a, b, c RADICI DI P

$a^2+b^2+c^2 = ?$

$$P(x) = (x-a)(x-b)(x-c)$$

$$= x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc$$

$$\begin{cases} a+b+c = 5 \\ ab+bc+ca = 3 \\ abc = -8 \end{cases}$$

$$(a+b+c)^2 = \boxed{a^2+b^2+c^2} + 2(ab+bc+ca)$$

$$a^2+b^2+c^2 = (5)^2 - 2\cdot 3 = 25-6 = 19$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{ab+bc+ca}{abc} = \frac{3}{-8}$$

$P(x) = x^3 - 6x^2 + 12x - 15$

a, b, c RADICI DI $P(x)$

$P(a) = 0 \quad P(b) = 0 \quad P(c) = 0$

TROVA POLI $q(x)$ GRADO 3 CON RADICI $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$

$$q(x) = x^3 \cdot \underbrace{P\left(\frac{1}{x}\right)} = \left[\left(\frac{1}{x}\right)^3 - 6\left(\frac{1}{x}\right)^2 + 12\left(\frac{1}{x}\right) - 15\right] x^3$$

$= 0$

$q(b+1) =$ — — —

OPPURE

$$\begin{cases} a+b+c = 6 \\ ab+bc+ca = 12 \\ abc = 15 \end{cases}$$

$A = -[a+1+b+1+c+1] = -(6+3)$

$B = (a+1)(b+1) + (b+1)(c+1) + (c+1)(a+1)$

$C = (a+1)(b+1)(c+1)$

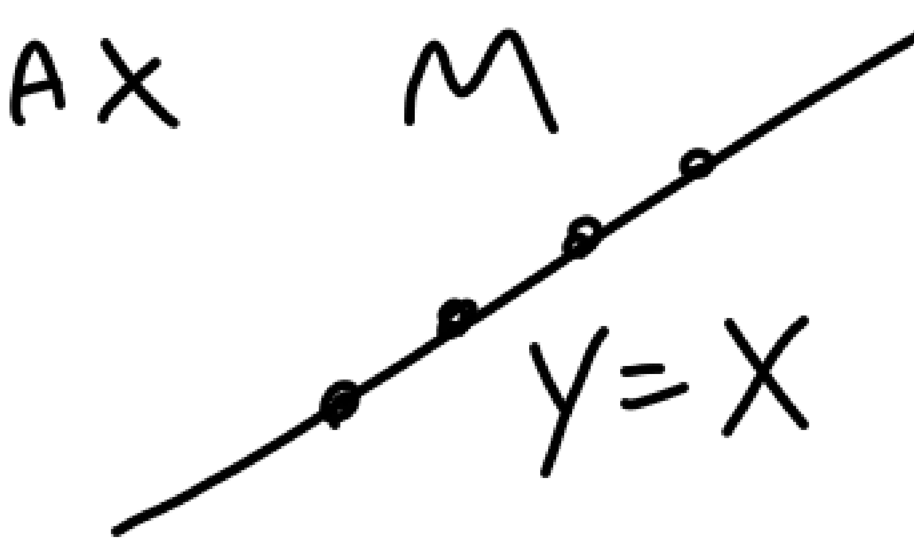
INTERPOLAZIONE

TUTTE $x_i \neq$

DATI $m+1$ PUNTI (x_i, y_i) SUL PIANO

ESISTE UNICO UN POLINOMIO DI GRADO AL MAX m

CHE PASSA PER TUTTI I PUNTI



es: POLINOMIO 4° GRADO MONICO $(-1;-1)$ $(0;0)$ $(1;1)$ $(2;2)$

$P(-1) = -1$

$P(0) = 0$

$P(1) = 1$

$P(2) = 2$

COSTRUISCO UN POLINOMIO AUSILIARIO:

$\boxed{q(x) = x}$ LA SOLUZIONE A "OCCHIO"

$d(x) = p(x) - q(x)$

$\deg(d) = 4$

MONICO

$d(-1) = p(-1) - q(-1) =$

$= -1 - (-1) = 0$

$$d(x) = (x+1)\cdot x \cdot (x-1)(x-2)$$

$$p(x) = d(x) + q(x)$$

$$= (x+1)x(x-1)(x-2) + x$$

$$p(3) = 4\cdot 3\cdot 2\cdot 1 + 3 = 27$$

$P(x)$ GRADO 3 NON NECESSARIAMENTE MONICO

$$\boxed{\begin{aligned} P(-1)&=1 \\ P(0)&=0 \\ P(1)&=1 \end{aligned}}$$

$P(2)=3$

PER TROVARE LA COSTANTE

$$3=P(2)=c\cdot 3\cdot 2\cdot 1+2^2$$

$$6c=3-4$$

$$c=-\frac{1}{6}$$

$q(x)=x^2$ "A OCCHIO"

$d(x)=P(x)-q(x)$ $\deg(d)=3$

$$d(-1)=P(-1)-q(-1)=1-1=0$$

$$d(0)=P(0)-q(0)=0-0=0$$

$$d(1)=P(1)-q(1)=1-1=0$$

$$d(x)=c\cdot(x+1)\cdot x\cdot(x-1)$$

$$P(x)=d(x)+q(x)=c(x+1)x(x-1)+x^2=-\frac{1}{6}(x+1)x(x-1)+x^2$$